

### 3. domaća matica

MMR

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$$\textcircled{1} \quad A + \lambda \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \in P(V) ; \quad A = P \cap g$$

P, g prenica:

$$\begin{aligned} p: ax+by+cz=0 &\Rightarrow x+\frac{b}{a}y+\frac{c}{a}z=0 \\ g: dx+ey+fz=0 &\Rightarrow x+\frac{e}{d}y+\frac{f}{d}z=0 \end{aligned}$$

$$x = -\frac{b}{a}y - \frac{c}{a}z =$$

$$\begin{aligned} &= z \frac{b}{a} \frac{da-cd}{ea-bd} - \frac{c}{a} z = \\ &= z \left( \frac{b}{a} \frac{da-cd}{ea-bd} - \frac{c}{a} \right) \end{aligned}$$

$$-\frac{b}{a}y - \frac{c}{a}z + \frac{e}{d}y + \frac{f}{d}z = 0$$

$$y \left( \frac{e}{d} - \frac{b}{a} \right) + z \left( \frac{f}{d} - \frac{c}{a} \right) = 0$$

$$\begin{aligned} y &= \frac{\left( \frac{c}{a} - \frac{f}{d} \right)}{\left( \frac{e}{d} - \frac{b}{a} \right)} z = \frac{\frac{cd-fa}{ad}}{\frac{ea-bd}{ad}} z = z \cdot \frac{cd-fa}{ea-bd} \\ z &= \frac{cd-fa}{ea-bd} \end{aligned}$$

$$A = \left( z \left( \frac{b}{a} \frac{da-cd}{ea-bd} - \frac{c}{a} \right), z \cdot \frac{cd-fa}{ea-bd}, z \right)$$

A lako predstavimo u homogenim koordinatama. Za  $z \neq 0$  je lako iznositi poljubno IR interval.

\textcircled{2}

pdim = 5, P, g, n prenica

$$p \cap g = \emptyset$$

$$p \cap n = \emptyset$$

$$g \cap n = \emptyset$$

$$\exists = 1 \quad p \cap n \neq \emptyset \wedge g \cap n \neq \emptyset \wedge n \cap g \neq \emptyset$$

z + prenica je pdim 3 ili 2 uektori, nek. proizvodi (z minus)  
ki predstavlja dve točki je pdim.

$$p: \exists p_1 \perp p_2$$

$$g: \exists g_1 \perp g_2 \mid \{g_1, g_2\} \perp \{p_1, p_2\}$$

$$n: \exists n_1 \perp n_2 \mid n_1 \perp \{g_1, g_2\}, n_2 \perp \{p_1, p_2\}$$

Ker je dim prostora 5  $\Rightarrow n_2$  lako iznositi.

$$n_2 = \underbrace{\alpha p_1 + \beta p_2}_{\downarrow} + \underbrace{\gamma g_1 + \delta g_2}_{\downarrow} + \eta n_1$$

$$n_2 = \alpha p + \gamma g \mid \alpha, \gamma \in \mathbb{R}, \alpha \neq 0$$

$$\exists \text{ prenica } n = \{(\alpha p, \gamma g) \mid \begin{cases} \alpha \cap p \neq \emptyset, \text{ ker } \alpha p \in P \cap p \\ \alpha \cap g \neq \emptyset, \text{ ker } \gamma g \in g \cap g \\ \alpha \cap n \neq \emptyset, \text{ ker } \alpha p \in n \end{cases}$$

Da  $\exists$  mth. 1 premica s: ("isto" kot na predavanjih za premico skup:  $p, g, A$ )

$$n_2' = n_p' + n_g'$$

$$n_2 = n_p + n_g$$

$$\Delta n_2 = n_2' = \underbrace{n_p'}_{\epsilon p} + \underbrace{n_g'}_{\epsilon g}$$

$$\Delta n_p + \Delta n_g$$

$$\underbrace{\Delta n_p - n_p'}_{\epsilon p} = \underbrace{n_g' - \Delta n_g}_{\epsilon g}$$

$$\text{ker } p \cap g = \emptyset \Rightarrow \Delta n_p - n_p' = 0$$

$$n_g' - \Delta n_g = 0$$

$$n_p' = \Delta n_p$$

$$n_g' = \Delta n_g$$

$$n = \mathcal{L}\{n_p, n_g\} = \mathcal{L}\{n_p', n_g'\}$$

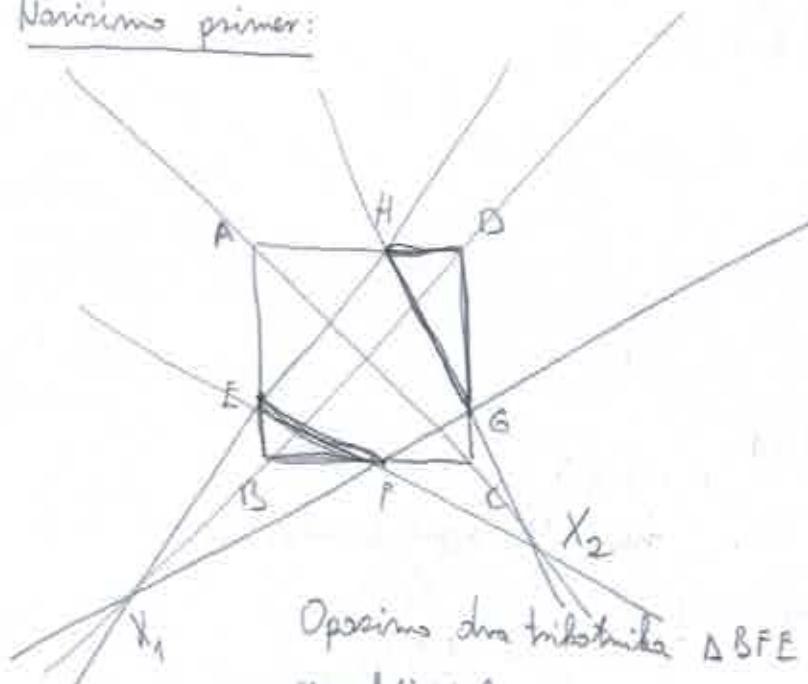
■ mth. 1 premic



$\Rightarrow$  zapis enolicen



3.

Naravimo pravac:

Oprezimo dva trikotnika  $\triangle BFE$  in  $\triangle HGD$ , ki nima npraktični legi.

Uporabimo Desarguesov teorem, ki pravi:

$$\exists \triangle BFE \wedge \exists \triangle HGD \text{ p.p. } \overline{EH} \cap \overline{BD} \cap \overline{FG} = X_1$$

$\Downarrow$

pravice

$$\overline{EB} \cap \overline{HD} = A$$

$$\overline{BF} \cap \overline{DG} = C$$

$$\overline{EF} \cap \overline{HG} = X_2$$

$\Downarrow$

$A, C, X_2$  kolinearni

$\overline{EF}, \overline{AC}, \overline{GII}$  se nekajjo in eni točki

Vemo: Če dve množini pravnic ne napoljuvajo množino eni točki,

$$\text{Naj bo } X_2 = \overline{EF} \cap \overline{HG}. \quad (1)$$

Po zgornjim izdelan skledi, da  $A, C, X_2$  kolinearni.

$$\Downarrow \quad X_2 \in \overline{AC} \quad (2)$$

$$(1), (2) \Rightarrow \overline{EF} \cap \overline{AC} \cap \overline{HG} = X_2$$

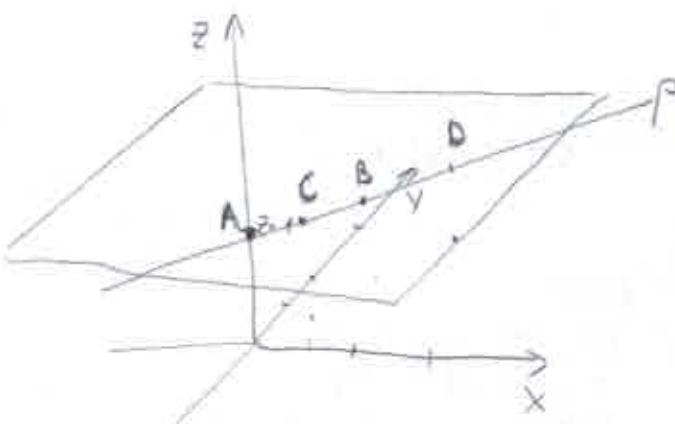
□

4.

$$\left. \begin{array}{l} A = [a] \\ B = [b] \\ C = [c] \\ D = [d] \end{array} \right\} \text{kolinearni } \sim P(\mathbb{R}^2)$$

Dokaz:  $a+b=c$   
 $\lambda a+b=d$ , za mrežu  $\lambda$

Definicija projektivne ravni na  $\mathbb{P}^1$ .



Imenovanje (homogene koordinate)

$$A = (1, 0, 1)$$

$$B = (1, 1, 1)$$

$$C = (1, 1, 2) = \left(\frac{1}{2}, \frac{1}{2}, 1\right)$$

$$D = (1, 1, 3) = \left(\frac{1}{4}, \frac{1}{4}, 1\right)$$

Imp:  $\lambda = -0.5$

$$D = (2, 2, 1)$$

Nosičimo pravou, da A, B, C, D ležijo na projektivni premici.

b)  
 $A = [-1, 0, 0]$   
 $B = [0, 1, 1]$   
 $C = [1, 2, 2]$   
 $D = [3, 2, 2]$

A, B, C, D kolinearni  
 Prvi mrežni

$$\begin{aligned} & C, D \text{ so mrežna in niti } \Rightarrow \vec{AB} \\ & -a=0 \\ & b+c=0 \\ & a+2b+2c=0 \\ & 3a+2b+2c=0 \end{aligned}$$

$$\begin{cases} a=0 \\ b=-c \end{cases}$$

$$0 \cdot x + by + cz = 0$$

$$by - bz = 0$$

$$b(y+z) = 0 \Rightarrow \text{Enačba proj. ravni je kolinearna}$$

Koordinatni vektori:

$$\begin{aligned} A &= [a] = [1, 0, 0] \\ B &= [b] = [0, 1, 1] \\ C &= [c] = [1, 2, 2] \\ D &= [d] = [3, 2, 2] \end{aligned}$$

$$A + B = [1, 0, 0] + [0, 1, 1] =$$

$$[1, 0, 0] + [0, 2, 2] = [1, 2, 2] = C$$

$$\lambda A + B = [1, 0, 0] + [0, 2, 2] = [1, 2, 2] = [3, 2, 2] = D$$

$$\boxed{\lambda = 3}$$

## Družji močni (2x kolinearni)

Ker je: A, B, C kolinearni in B, C, D kolinearni  $\Rightarrow$  A, B, C, D kolinearni

Tačke (a, b, c), (d, e, f), (g, h, i) so kolinearni, ker  $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 0$

A, B, C, D - kolinearni

$$A \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = (-1) \cdot (2 - 2) = 0$$

$$B \begin{vmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 3 & 2 & 2 \end{vmatrix} = (-1)(2 - 6) + 1 \cdot (2 - 6) = 4 - 4 = 0$$

$\Rightarrow A, B, C, D$  kolinearni

c)

$$\lambda = \frac{(c-a)(d-b)}{(c-b)(d-a)} = \frac{\overrightarrow{AC} \cdot \overrightarrow{BD}}{\overrightarrow{BC} \cdot \overrightarrow{AD}} = \text{mnz.-ratio}(AB, CD) = \frac{\beta\gamma}{25} \stackrel{\beta\gamma = 1}{=} \frac{1 \cdot \lambda}{1 \cdot 1} = \lambda \quad \square$$

$$\begin{aligned} c &= \lambda a + \beta b \\ d &= \gamma a + \delta b \\ \lambda &= \beta = \gamma = 1 \\ \gamma &= \lambda \end{aligned}$$